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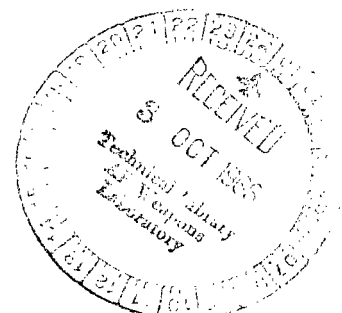


**COMPARISON OF EXPERIMENTAL AND  
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MISMATCH IN A CIRCUMFERENTIAL JOINT  
IN A CYLINDRICAL PRESSURE VESSEL**

*by William C. Morgan and Peter T. Bizon*

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# COMPARISON OF EXPERIMENTAL AND THEORETICAL STRESSES AT A MISMATCH IN A CIRCUMFERENTIAL JOINT IN A CYLINDRICAL PRESSURE VESSEL

by William C. Morgan and Peter T. Bizon

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## SUMMARY

An experimental evaluation was made of a theoretical analysis of the elastic stress distribution at a mismatched circumferential junction between thin-walled, internally pressurized cylinders (nominal stress field, 2 to 1). The test cylinders used in the investigation were of uniform wall thickness but had dissimilar radii on each side of the junction. The nominal ratio of radius to wall thickness was 75.

Stress distributions were determined with electric-resistance strain gages which were mounted on the interior and exterior walls of four test cylinders, each with a different mismatched junction. Three cylinders were provided with tapered-fillet discontinuities of various degrees of mismatch; the fourth cylinder was made with an abrupt discontinuity at the mismatched junction.

A correlation between test results and an analytically predicted stress distribution was established for each case.

## INTRODUCTION

An experimental investigation was conducted to evaluate a theoretical procedure (ref. 1) for prediction of the elastic stress distribution in the region of a mismatch at the circumferential junction between thin-walled internally pressurized cylinders (nominal stress field, 2 to 1).

This type of mismatch may occur when a number of cylinders must be joined by welding to provide a long tank structure characteristic of pressure-bottle or space-vehicle design. An appreciable percentage of mismatch may result from a quantitatively small middle-surface discontinuity at a junction in a thin-shelled structure. In this report, a middle surface is defined as the surface generated by rotating a meridian midway through the wall thickness of a cylinder about the axis.

The case for such a discontinuity with negligible fillets at the junction has been considered in other investigations (refs. 2 and 3). The more recent study of reference 1 essentially is a further development of that analysis and includes consideration of the effect of tapered fillets.

The purpose of the present investigation was to provide experimental verification of the theoretical analysis. In that part of the analysis applicable to the specific configuration under consideration, the principal simplifying assumptions were that radial symmetry existed, that there would be no effect of local stress concentration, and that the relation between pressure and strain would be linear.

The scope of the investigation included tests of four cylindrical specimens, each of which was machined with a different mismatch at a circumferential junction, all with nominal ratios of radius to wall thickness of 75. Three of the specimens had 45° tapered fillets at their junctions and were made with discontinuities of 20, 60, and 100 percent, respectively, based on the radial difference between middle surfaces compared to wall thickness. The junction of the fourth cylinder incorporated an abrupt-discontinuity mismatch of 60 percent. This cylinder was included to evaluate the analysis of reference 1 as applied to a junction with negligible fillets.

Stress distributions were determined experimentally from strain-gage data and compared to theoretical values. All data were obtained in the elastic range of the specimen material.

## THEORETICAL ANALYSIS

This section does not describe in detail the analyses for the configurations tested in the experimental investigation; the theoretical studies are presented in full in references 1 to 3. However, a brief outline of the methods used to determine stress distribution might prove of interest in this report.

In the analysis that included the effect of tapered fillets, the structure is considered as separated into three parts, the filleted junction (considered as a rigid ring of uniform cross section) and two individual cylinders, as shown in figure 1(a). Under pressure, radial expansion will be dissimilar among these components, and the maintenance of structural integrity will impose shear forces and bending moments between the ring and each of the cylinders. The loads on the ring can be resolved into radial forces and twisting couples uniformly distributed around the ring, and the cylinders can be considered as simple edge-loaded cylinders. The shears and moments may be evaluated by equating the slopes and displacements at the ring-cylinder junctions and then by obtaining simultaneous solutions of the four equations. Substitution of these values into equations from reference 4 provides a solution for that part of the stress distribution attributable to the

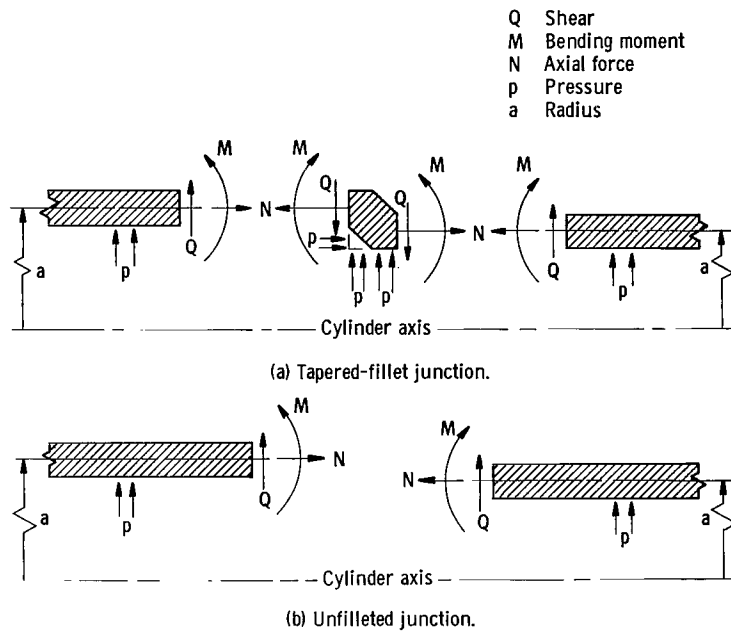


Figure 1. - Theoretical analysis of tapered-fillet junction and unfilleted junction.

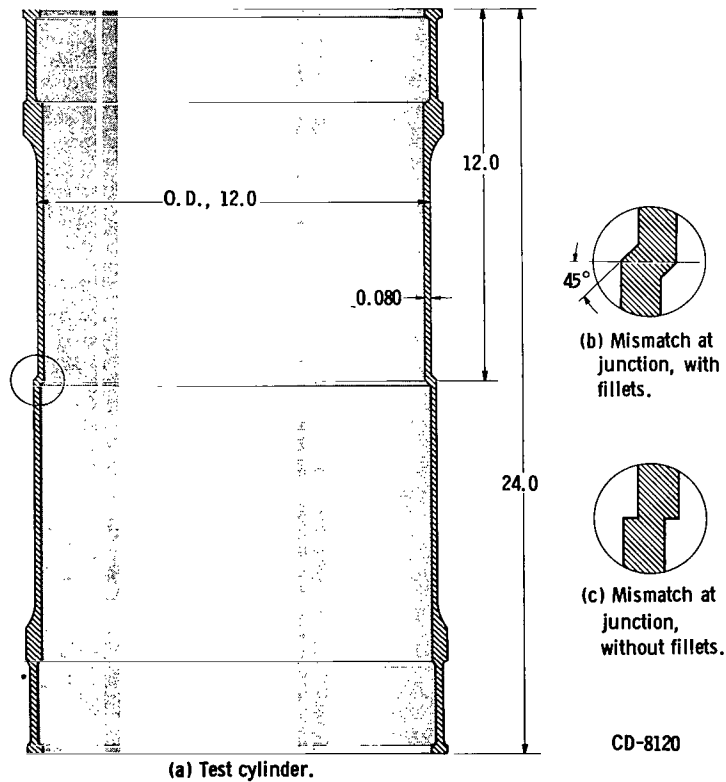


Figure 2. - Test cylinder configuration (dimensions in inches).

discontinuity. This stress distribution may be added to the membrane stresses for the final solution.

In the case of a cylindrical vessel with a circumferential mismatch and no fillets, the structure is considered as separated into two cylinders, as shown in figure 1(b). Stress distribution is determined essentially as described for the filleted mismatch; absence of the ring segment permits a less complicated solution.

## APPARATUS AND PROCEDURE

### Test Cylinders

The test cylinders were fabricated with a nominal ratio of radius to wall thickness of 75. Figure 2(a) shows the basic configuration of the cylinders used in the investigation. Three cylinders were made with circumferential junctions with mismatches of 20, 60, and 100 percent, respectively, based on the radial difference between middle surfaces compared to wall thickness, and the junctions were provided with  $45^\circ$  tapered fillets, as shown in figure 2(b). Values for percent of mismatch were selected arbitrarily to provide a range of mismatch and were not intended to be acceptable limits of error in space-vehicle assembly. A fourth cylinder was provided with a 60-percent mismatch and minimum fillet radii at the junction, as shown in figure 2(c).

The cylinders were machined from 2014-T6 extruded aluminum tubing. Cylinder-

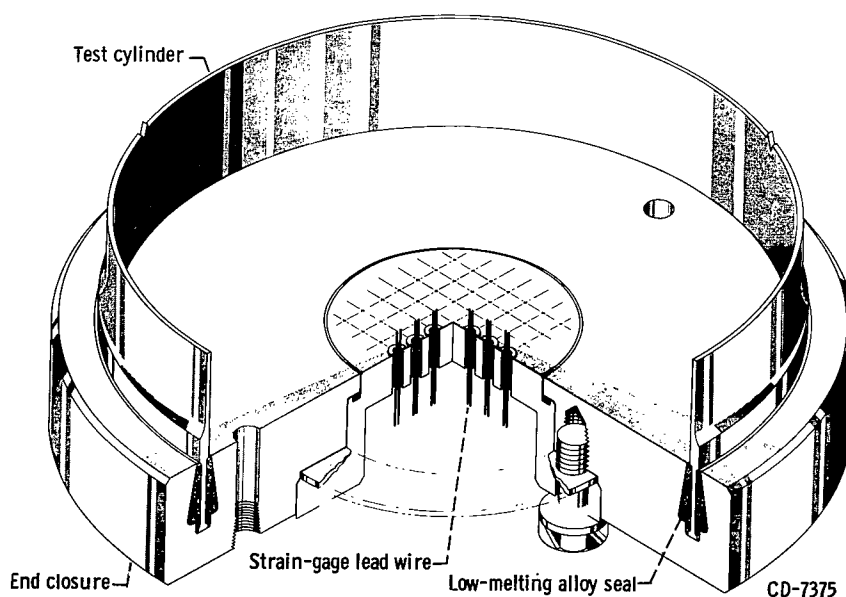


Figure 3. - End closure with method of bringing out lead wires from interior-wall strain gages.

wall thickness was  $0.080 \pm 0.001$  inch; diameters were within  $\pm 0.002$  of specified values, and out-of-roundness did not exceed 0.001 inch. End closures of the type shown in figure 3 were provided. Assembly and disassembly of these closures required that the parts be brought to the temperature required to liquefy the alloy that forms the seal ( $325^{\circ}$  F).

## Strain-Gage Installation

The nominal locations of strain gages on the inner and outer surfaces along a meridian are shown in figure 4. Two gages oriented to measure the principal strains were installed at each location on both the inner and outer walls. After the strain gages were installed, the exact locations were determined with precision measuring equipment. Selection of meridians for strain-gage mounting was arbitrary. Figure 5 shows two views of a typical installation. (Fig. 3 shows the method of bringing the lead wires out from the interior-wall strain gages.)

Strain measurements were made with 120-ohm electrical-resistance foil strain gages which were bonded with cyanoacrylate cement. The gages had effective lengths and widths of  $1/16$  inch. Lead wires were brought to a terminal box near the test facility and were connected through a five-wire system to a multichannel digital strain recorder. This equipment incorporated systems for balancing, calibrating, controlling, scanning, and recording strain-gage output to an accuracy of  $\pm 1.0$  percent.

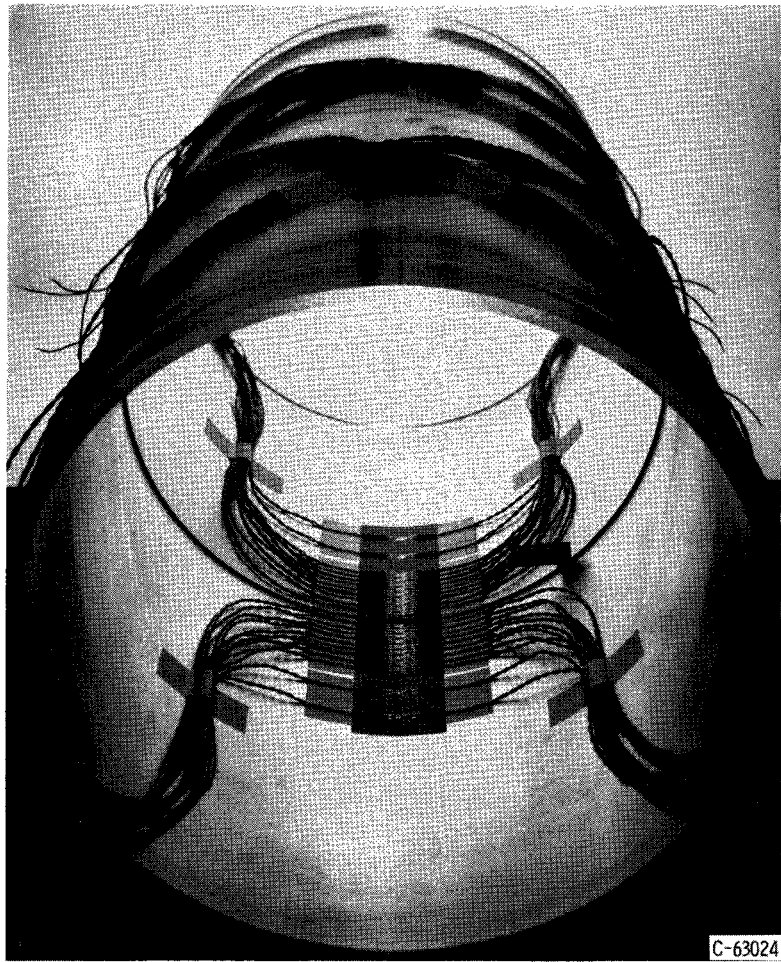


Figure 4. - Location of perpendicularly oriented strain gages along test-cylinder meridian.

## Test Procedure

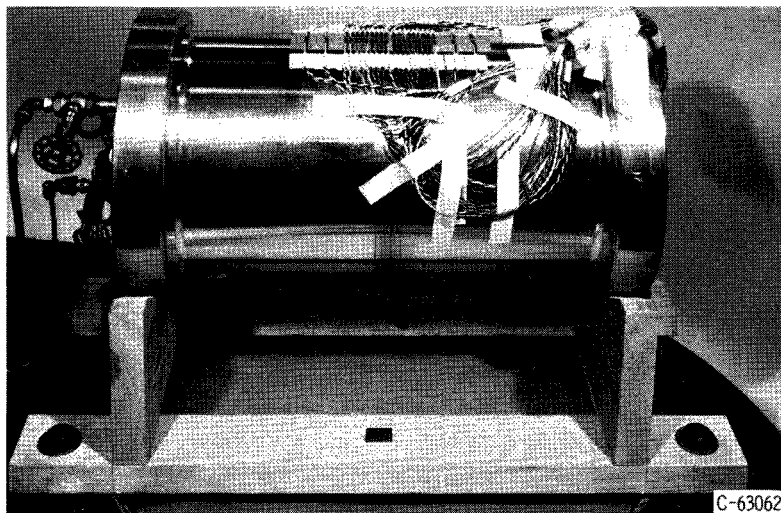
Figure 5(b) shows a cylinder in the test facility. The horizontal mounting was chosen to eliminate any meridional stresses that might arise from the presence of the relatively heavy end closures. In order to compensate for any error that might still occur because of bending, each cylinder was tested in three positions: with the strain-gage meridian upward, as shown in figure 5(b); with the cylinder rotated  $90^{\circ}$  from that position; and with the cylinder rotated  $180^{\circ}$  from the initial position. Data from the three tests were averaged.

Internal pressure was produced by nitrogen gas and measured by a pressure gage which was calibrated and found to have an error of less than 0.5 pound per square inch. The test procedure consisted of raising the internal pressure to a predetermined value



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(a) Strain gages mounted on interior wall.



C-63062

(b) Cylinder in test facility.

Figure 5. - Typical strain-gage installation.



and returning it to zero pressure according to a time schedule that provided sufficient intervals for stabilization of the strain gages. This basic cycle was repeated for progressively higher pressures until the maximum required pressure had been achieved. The purpose of making successive increases in pressure was to provide data for correction of any nonlinearity in strain-gage output. Data were recorded at pressures of 100, 110, 120, 130, 140, and 150 pounds per square inch, and duplicate tests were made at each of the three cylinder positions.

## Reduction of Data

The computer program used to reduce the data read in the strain-gage outputs, corrected for zero drift, corrected any nonlinearity by using the method of least squares, averaged the strains, and converted strains to principal stresses and effective stresses (yield criterion based on distortion-energy theory) by using the following relations (with the material assumed to be isotropic):

Meridional stress

$$\sigma_x = \frac{E}{1 - \nu^2} (\epsilon_x + \nu \epsilon_\theta) \quad (1)$$

Circumferential stress

$$\sigma_\theta = \frac{E}{1 - \nu^2} (\epsilon_\theta + \nu \epsilon_x) \quad (2)$$

Effective stress

$$\sigma_e = \sqrt{\sigma_x^2 + \sigma_\theta^2 - \sigma_x \sigma_\theta} \quad (3)$$

where

$\sigma$  stress, psi

$E$  modulus of elasticity, psi

$\nu$  Poisson's ratio, 1/3

$\epsilon$  strain, in./in.

and the subscripts  $x$ ,  $\theta$ , and  $e$  denote meridional, circumferential, and effective stresses, respectively.

## RESULTS AND DISCUSSION

The strain data were reduced to principal and effective stress values, as described in the section Reduction of Data, and these experimental results were compared with the stress distributions obtained by theoretical analysis. In order to provide a parameter useful in comparisons among the different cases tested, the stress values were divided by membrane stress computed on the basis of thin-wall theory, using an average radius for an individual cylinder.

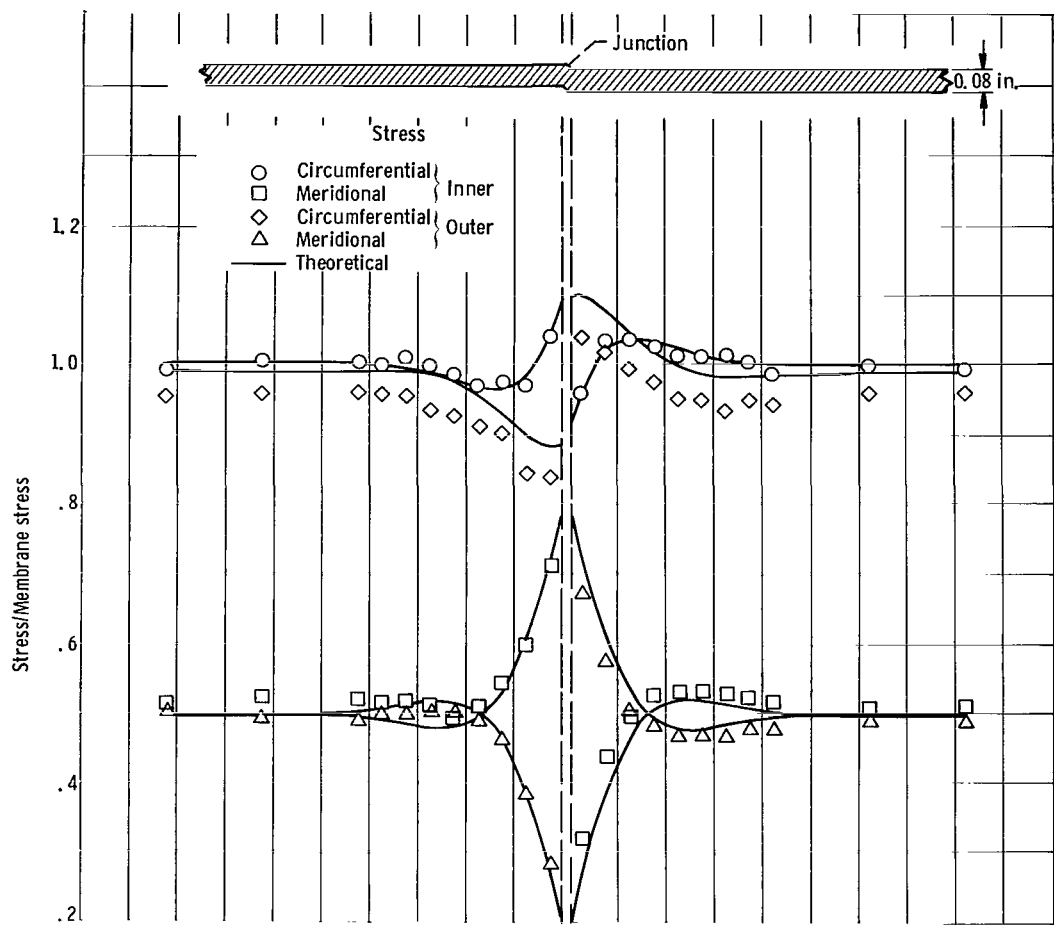
### Mismatch with Fillets

Comparisons are shown in figures 6, 7, and 8, for the tapered-fillet junctions with mismatches of 20, 60, and 100 percent, respectively. In general, there appeared to be good correlation.

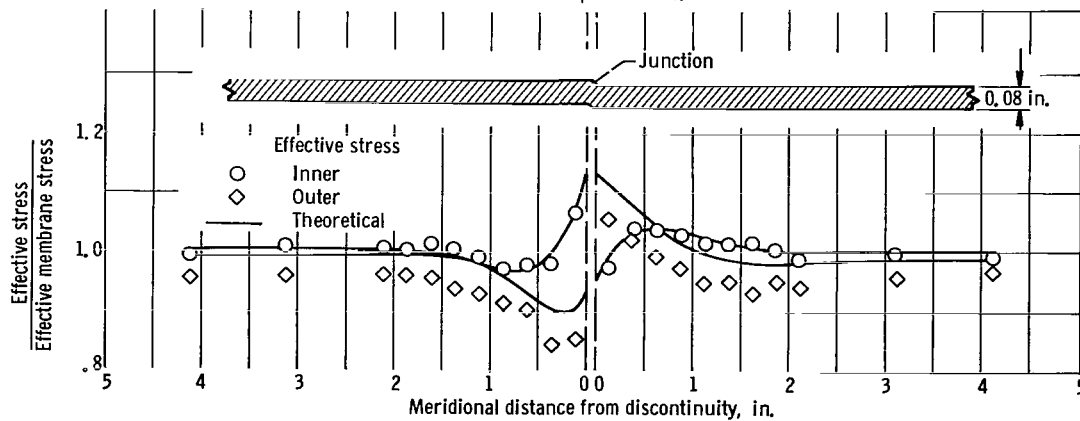
In the case of the 20-percent mismatch with fillets (fig. 6(a)), the meridional stresses determined by experiment were substantially in agreement with analytical values, and approximately the same degree of correlation was established for the inner-wall circumferential stress distribution. The most marked disparity between experimental and analytical values was observed for outer-wall circumferential stresses, as will be discussed in the section Experimental Error. On a qualitative basis the experimental results closely followed the trends predicted by theory. This is shown more clearly by the comparison of effective stress distributions in figure 6(b). The inner-surface stress values obtained experimentally were nearly coincident with analytical curves of stress distribution, and the experimental stress points for the outer surface varied from the analytical curve by a fairly constant value.

Figure 7(a) presents the principal stresses obtained from strain data compared with the analytical stress distribution for the 60-percent mismatch with fillets. In general, correlation was satisfactory, both quantitatively and qualitatively, although there was some disparity between the experimental values and the theoretical stress distribution for the inner-wall meridional and outer-wall circumferential stresses. The effect of this difference is shown in the comparison based on effective stress (fig. 7(b)). The test results tended to follow the theoretical curve for stress distribution, but considerable quantitative discrepancy occurred near the junction.

Comparison between test results and theoretical curves for the case of 100-percent

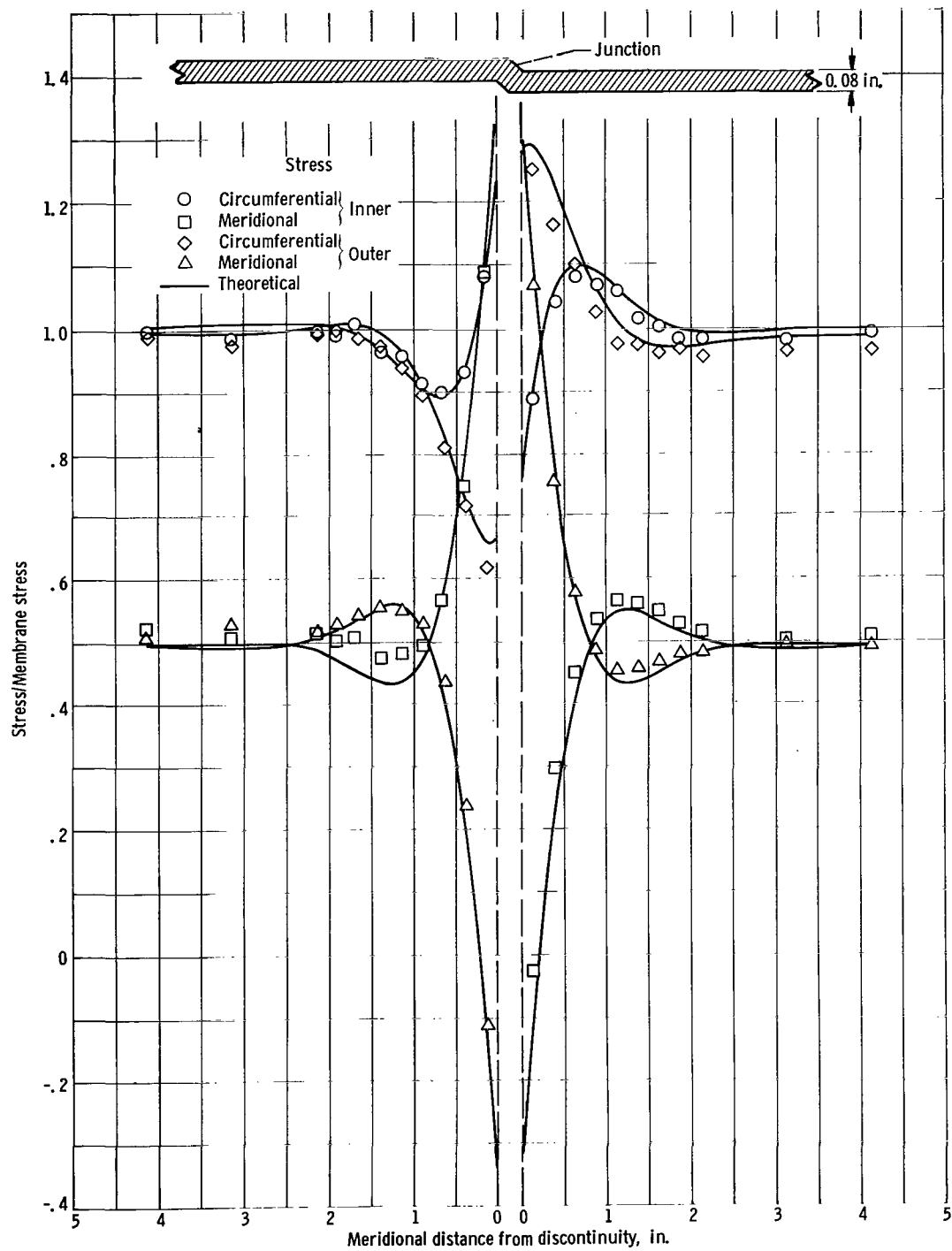


(a) Principal stresses.



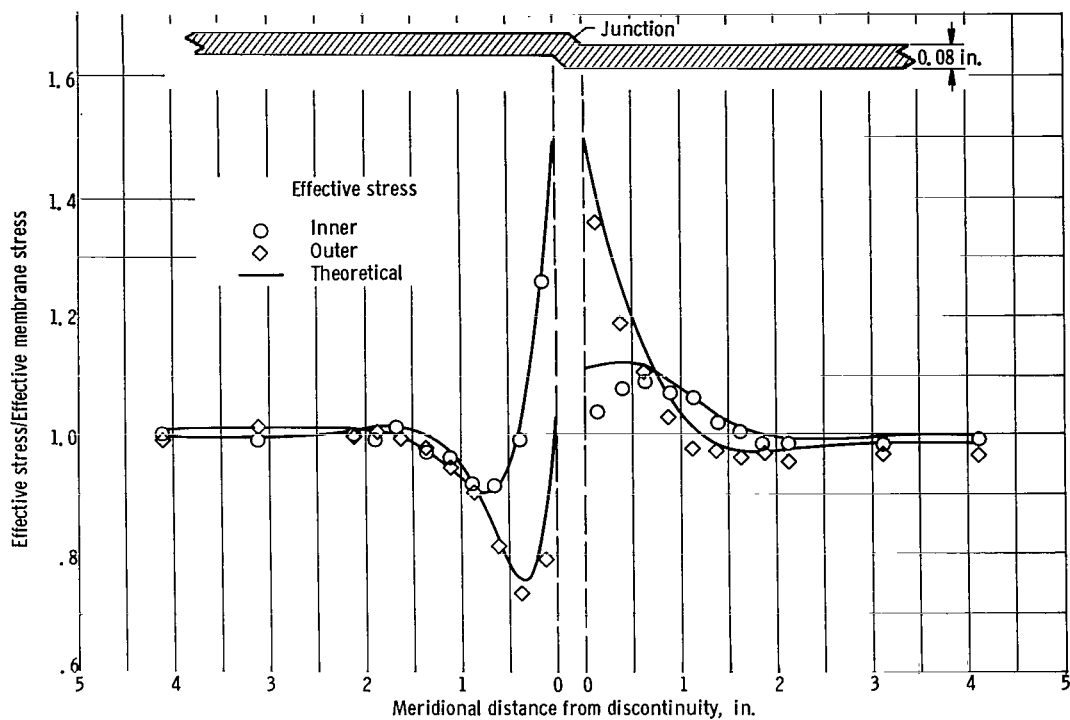
(b) Effective stresses.

Figure 6. - Theoretical and experimental stress distribution at tapered-fillet junction with 20 percent mismatch.



(a) Principal stresses.

Figure 7. - Theoretical and experimental stress distribution at tapered-fillet junction with 60 percent mismatch.



(b) Effective stresses.

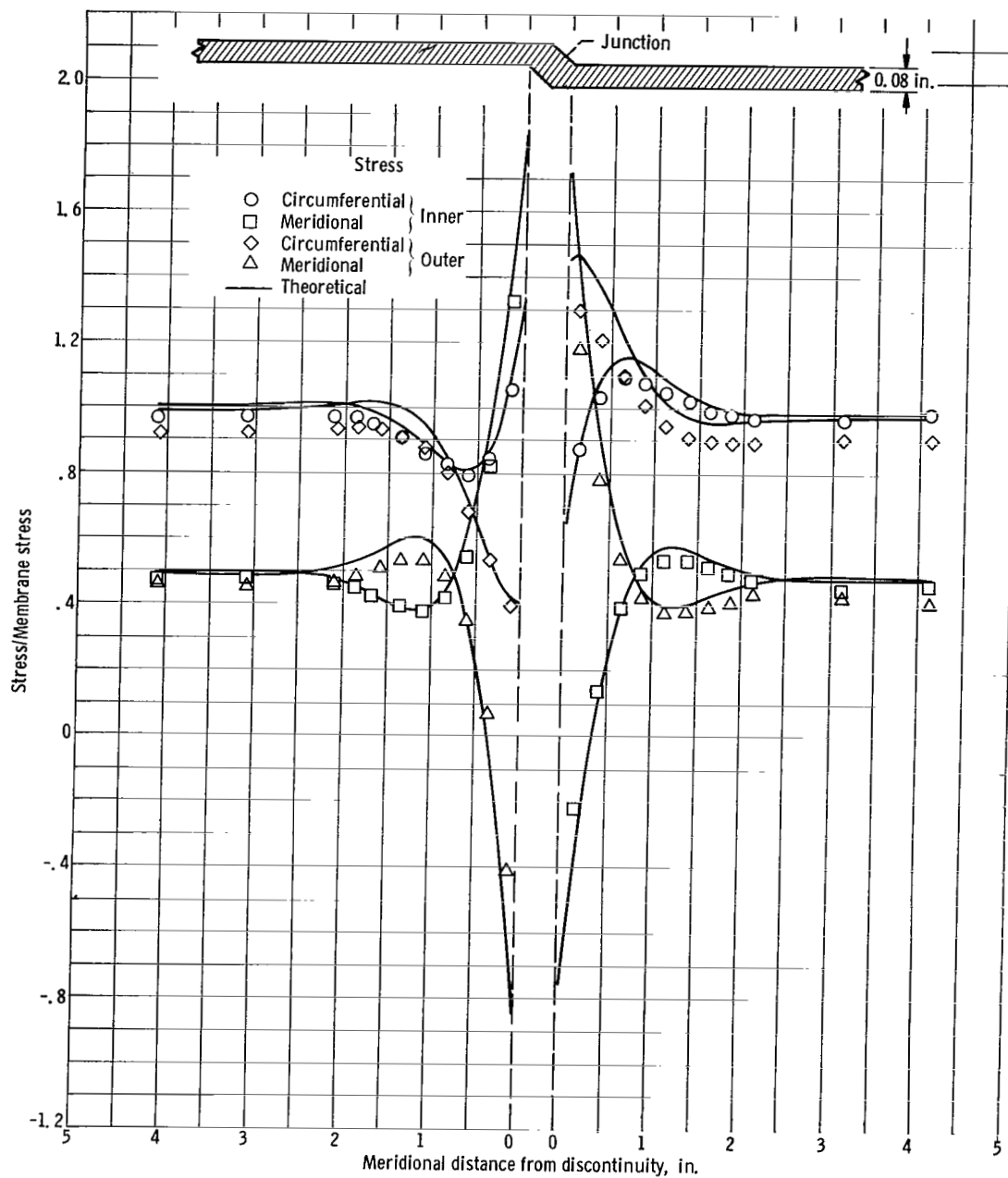
Figure 7. - Concluded.

mismatch with fillets confirmed the validity of the analysis qualitatively, as shown in figure 8(a), but the experimentally determined principal stresses tended to be consistently low, with the exception of a few points for meridional stress. This effect is exemplified more clearly in figure 8(b), which compares effective stresses.

### Mismatch at Abrupt Discontinuity

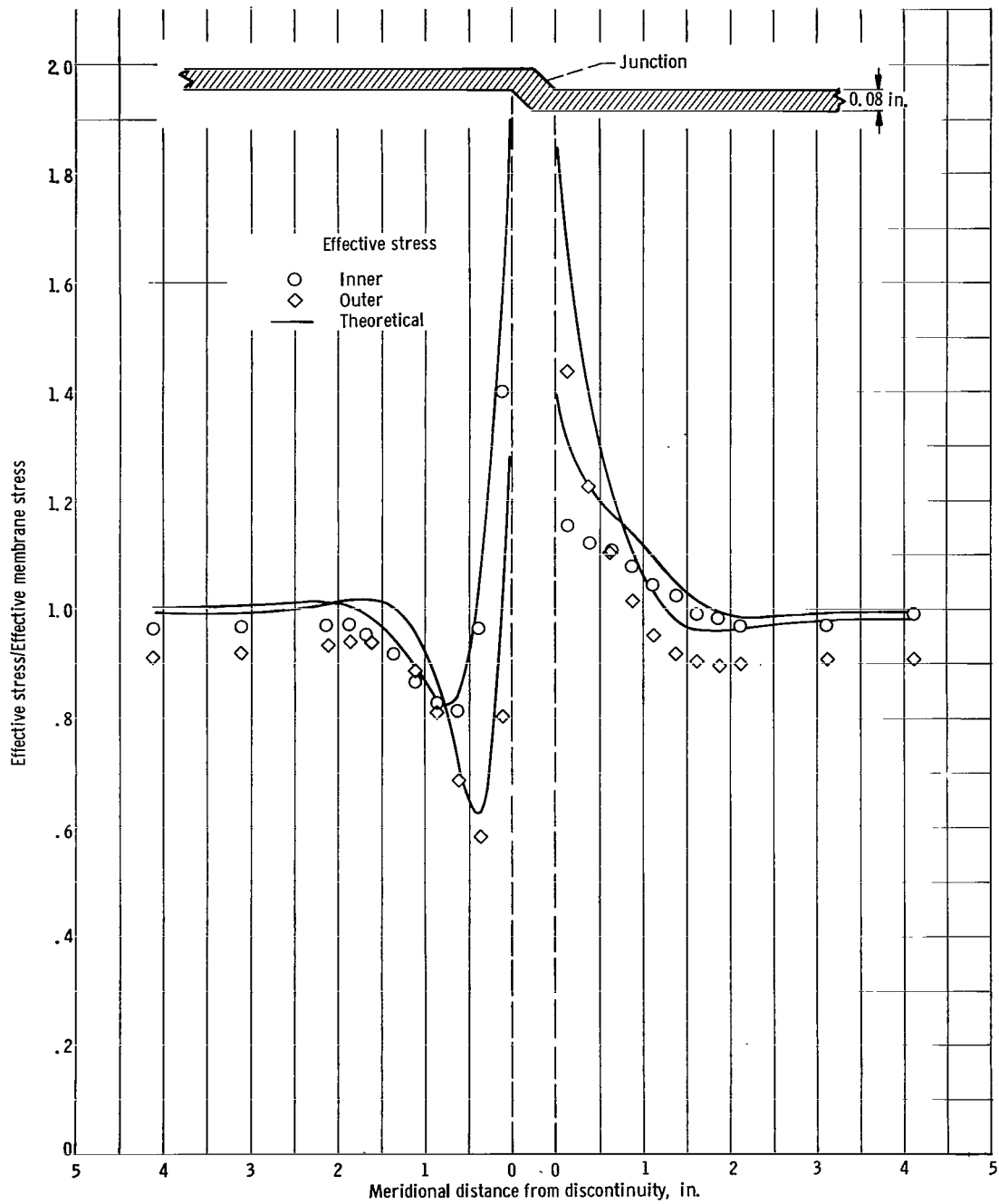
Figure 9(a) shows the degree of correlation between stresses based on experimental data and the analytical stress distribution for a 60-percent mismatch with negligible fillets. This comparison between observed and predicted principal stresses validated the analysis of reference 1 for the assumptions described on page 2 of this report. The comparison based on effective stress (fig. 9(b)) shows that the experimental results were consistently lower than the analytical values, but the disparity was not excessive, and the trend of the experimental stress points was in close agreement with the theoretical curves of stress distribution.

Comparison between figures 7 and 9, for 60-percent mismatch, with fillets and without fillets, respectively, shows that the presence of fillets made no significant difference in stress distribution. This result agrees with the same conclusion in reference 1 which



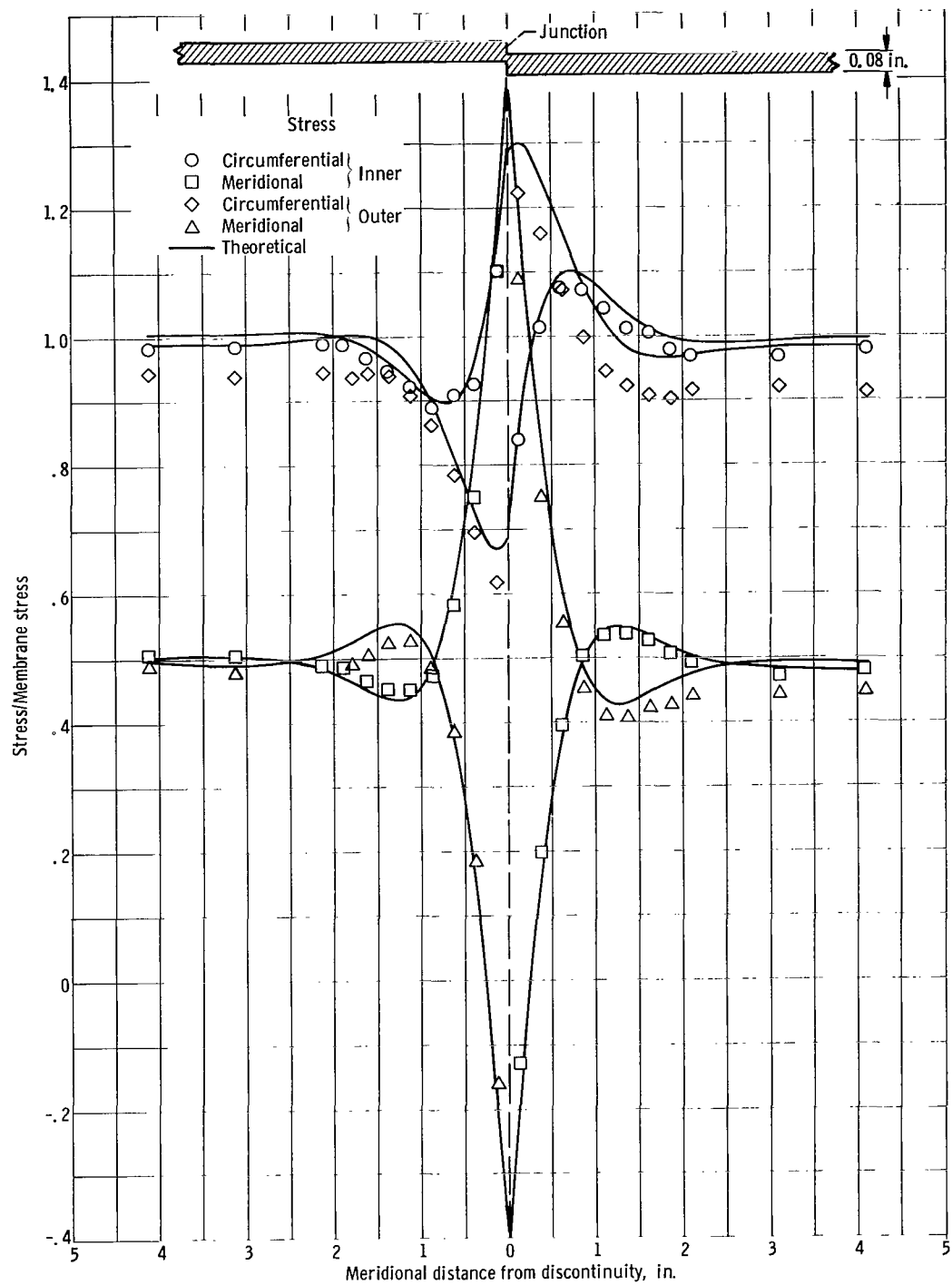
(a) Principal stresses.

Figure 8. - Theoretical and experimental stress distribution at tapered-fillet junction with 100 percent mismatch.



(b) Effective stresses.

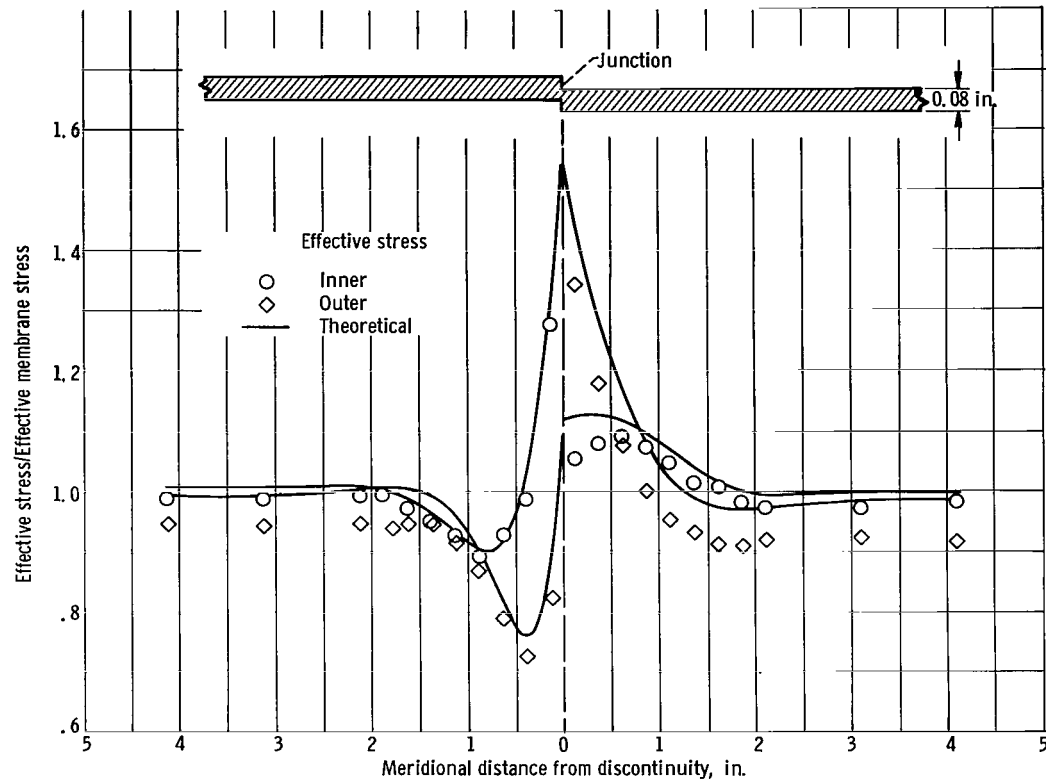
Figure 8. - Concluded.



(a) Principal stresses.

Figure 9. - Theoretical and experimental stress distribution at unfilleted junction with 60 percent mismatch.





(b) Effective stresses.

Figure 9. - Concluded.

indicated that an effect of considerable magnitude would be expected to occur only in a cylindrical structure of extremely large ratio of radius to wall thickness and under high stress.

## Experimental Error

The discrepancies between data points and corresponding theoretical stress values were determined as percentages of theoretical stress. The average for all cylinders tested was 5 percent, on the basis of absolute values.

Considered in terms of algebraic percentages, the most consistent error appeared to be in the data for outer-wall circumferential stress. These data were low to an extent not ascribable to fabrication or instrument error, as has been observed in another investigation (ref. 5). There is a possibility that the method of manufacture of the basic material of the test specimens may be the cause. The cylinders used in this investigation and in the tests described in reference 5 were made from extruded tubing; specimens of similar material machined from solid billets were used in the investigations reported in

references 6 and 7, and for these specimens the data were not characterized by low values for outer-wall circumferential stress.

It should be pointed out, however, that the possible presence of orthotropic material properties does not provide an adequate reason for the discrepancy between experimental and theoretical outer-wall circumferential stresses. An examination of the equations for an orthotropic stress field indicated that the experimentally determined meridional stresses also should have shown an appreciable effect, and this was not the case. No entirely satisfactory explanation can be advanced at the present time.

## CONCLUDING REMARKS

The investigation included tests of three tapered-fillet junctions with mismatches of 20, 60, and 100 percent, respectively, and one abrupt junction with a 60-percent mismatch. The results of the experimental investigation substantiated the theoretical analysis of reference 1 for the stress distribution in the region of a circumferentially mismatched junction between thin-walled cylinders under internal pressure.

Lewis Research Center,  
National Aeronautics and Space Administration,  
Cleveland, Ohio, May 19, 1966,  
124-11-06-01-22.

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